Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Problem Solving-II

Subject Code: 5SC03PRS1 **Branch: M.Sc.(Mathematics)**

Semester: 3 **Date : 06/12/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Generate a field of order 9. **(02)**

b. Write β^{99} in disjoint cycle form ,where $\beta = (1\ 2\ 3)(1\ 4\ 5)$. **(02)**

c. Suppose $\phi: z_{30} \to z_{30}$ is homomorphism and $\ker\phi = \{0, 10, 20\}$. If $\phi(23) = 9$ then determine all elements that map to 9. **(02)**

d. True or False : Any finite cyclic group is isomorphic to Z. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Determine the number of elements of order 5 in $z_{25} \oplus z_5$. **(06)**

b) Find all maximal ideal in $\boldsymbol{Z}_{12}$. **(04)**

c) Define: Conjugate Class. Also find the conjugate classes and class equation of Q_8. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) i. Check that $f(x) = x^2 + 312312x + 123123$ is reducible over Q ? State the result which you have use. **(06)**

ii. Compute $5^{15}\ mod\ 7$ and $7^{13}\ mod\ 11$. Also state result which you use.

b) How many homomorphism are there from z_{20} to z_{10} ? List all homomorphism. **(04)**

c) Find all units of $J[i]$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Solve the given equation by Gauss-Seidel method **(06)**

$$2x + y + z = 4,\ \ x + 2y + z = 4,\ \ x + y + 2z = 4.$$

b) Given $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 200, y_4 = 100$. Find $\Delta^4 y_0$ without forming the difference table. **(04)**

c) If $O(G) = 10$, then how many 2-Sylow subgroup and 5-Sylow subgroups in G. Which of them are normal? **(04)**

OR

Q-3 **Attempt all questions** **(14)**

a) Find a cubic polynomial which takes the following set of values using Newton's forward method. **(06)**

x	0	1	2	3
y	1	2	1	10

b) Show that the set Q \ {-1} is an abelian group with respect to the binary operation $a*b=a+b+ab, \forall a,b \in G$. **(04)**

c) i) If a is an element of group G and $|a| = 7$, show that a is a cube of some element of G. **(04)**

ii) Let $H = \left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a, b, d \in R, ad \neq 0\right\}$. Is H normal subgroup of $GL(2: R)$?

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define : Boolean Ring and give an example of it. **(02)**

b. Find particular integral for $(D^2 + DD' - 6D'^2)z = x + y$ **(02)**

c. Classify the following partial differential equations: **(02)**

i. $2u_{xx} + 4u_{xy} + 3u_{yy} = 2$

ii. $u_{xx} + 4u_{xy} + 4u_{yy} = 0$

d. Evaluate : $\Delta \cos x$ **(01)**

Q-5 **Attempt all questions** **(14)**

a) Find the integral surface of the partial differential equation $(x-y)p + (y-x-z)q = z$, passing through the circle $z = 1$, $x^2 + y^2 = 1$. **(06)**

b) Solve:$xp + yq = pq$ **(04)**

c) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$ **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a) Using Newton's divided difference formula evaluate $f(8)$, given that **(07)**

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

b) Solve : $z^2(p^2 + q^2 + 1) = 1$. **(04)**

c) Find the characteristics of $4u_{xx} + 45 + 3u_{yy} + u_x + u_y = 2$ **(03)**

Q-6 **Attempt all questions** **(14)**

a) Using Picard's method find $y(0.2)$ given that $y' = x - y; y(0) = 1$ and $h = 0.1$ **(05)**

b) Solve: $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$. **(05)**

c) Consider the initial value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(0,y) = 4e^{-2y}$ then find $u(x,y)$ and $u(1,1)$. **(04)**

OR

Q-6 **Attempt all Questions**

a) Using Runge-Kutta method of fourth order solve for $y(0.1), y(0.2)$ given that $y' = xy + y^2;\ y(0) = 1$. **(07)**

b) By using method of separation of variable solve two dimensional Laplace equation **(07)**